

2020 17.30 ~~수업시간~~

1. Find solutions for

$$\begin{cases} x \equiv 5 \pmod{9} \\ x \equiv 7 \pmod{14} \\ x \equiv 21 \pmod{35} \end{cases}$$

2. prove or disprove:

Let p be an odd prime.

$$x^4 \equiv -1 \pmod{p} \xrightarrow{\text{is solvable}} p \equiv 1 \pmod{4}$$

3. prove or disprove:

$$\phi(2n) = \phi(n) \iff n: \text{odd}$$

4. Find a basis for $N(A)$ where

$$A = \begin{pmatrix} 2 & 3 & 4 & 5 & 7 \\ 4 & 6 & 8 & 10 & 11 \\ 6 & 8 & 12 & 15 & 21 \end{pmatrix}$$

5. Discuss $A = (a_{ij})_{4 \times 4}$ where $r(A) = 4$.

6. Derive two applications from BETTMAN Extension Theorem.

7. Find

$$\textcircled{1} \{ \phi \mid \phi: (\mathbb{Q}, +) \rightarrow (\mathbb{Q}, +): \text{ } \forall p \text{ homo} \}$$

$$\textcircled{2} \{ \phi \mid \phi: (\mathbb{Z}_4, +, \cdot) \rightarrow (\mathbb{Z}_2, +, \cdot): \text{ ring homo} \}$$

8. prove or disprove:

Let $|G| = 14$.

$$\textcircled{1} H \leq G, |H| = 7 \implies H \trianglelefteq G$$

$$\textcircled{2} \left(\begin{array}{l} K_i \leq G, |K_i| = 2, K_i \leq G \\ L \leq G, |L| = 2 \end{array} \right) \implies \bigcap_{i \in I} K_i \trianglelefteq G$$

$$L = K_i \text{ for some } i$$

9. Discuss the Abelianization $A(G)$ of a group G .